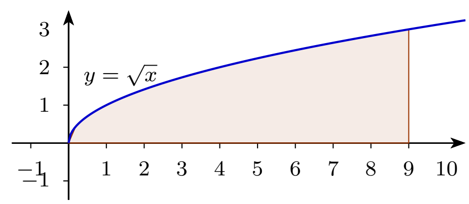
MATH2020A Homework 2

(15.2)

13. Sketch this region,



Then

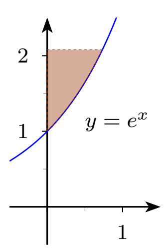
(a) Vertical cross-sections

$$\iint\limits_R dA = \int_0^9 \int_0^{\sqrt{x}} dydx$$

(b) Horizontal cross-sections

$$\iint\limits_R dA = \int_0^3 \int_{y^2}^9 dxdy$$

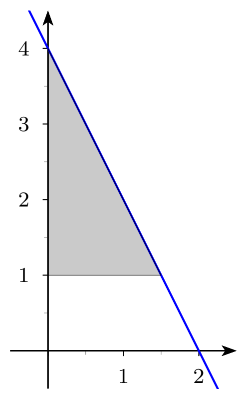
21. Sketch the region for $0 \leq x \leq \ln y$ and $1 \leq y \leq \ln 8$



Then

$$\int_1^{\ln 8}\int_0^{\ln y} e^{x+y}dxdy = \int_1^{\ln 8} e^y(e^{\ln y}-1)dy = \int_0^{\ln 8} e^y(y-1)dy$$
$$= (\ln 8-2)e^{\ln 8}-(1-2)e = 24\ln 2 - 16 + e$$

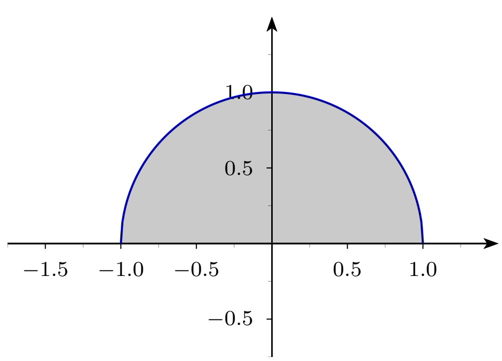
32. Sketch the region



Then

$$\int_0^{\frac{3}{2}}\int_1^{4-2u}\frac{4-2u}{v^2}dvdu = \int_0^{\frac{3}{2}}(-\frac{4-2u}{4-2u}+\frac{4-2u}{1})du = \int_0^{\frac{3}{2}}(3-2u)du$$
$$= 3\times\frac{3}{2}-\frac{9}{4}=\frac{9}{4}$$

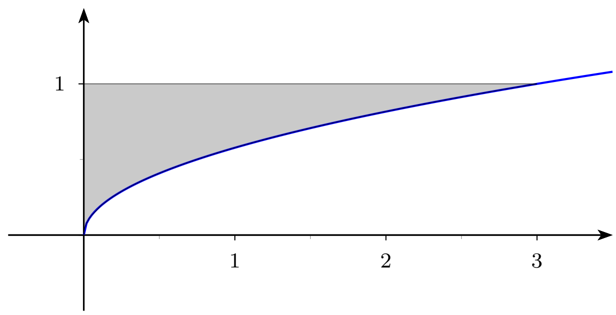
41. Sketch the region



Then change the order of integration

$$\int_0^1\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} 3ydxdy = \int_{-1}^1\int_0^{\sqrt{1-x^2}} 3ydydx$$

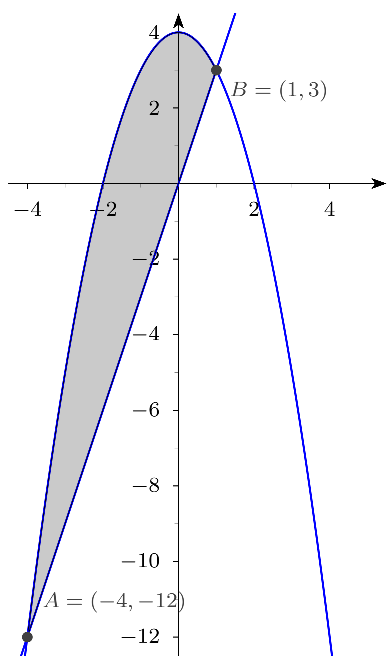
52. Sketch the region,



Then reverse the order of integration

$$\int_0^3 \int_{\sqrt{x/3}}^1 e^{y^3} dydx = \int_0^1 \int_0^{3y^2} e^{y^3} dxdy = \int_0^1 3y^2 e^{y^3} dy$$
$$= \left[e^{y^3}\right]_0^1 = e - 1$$

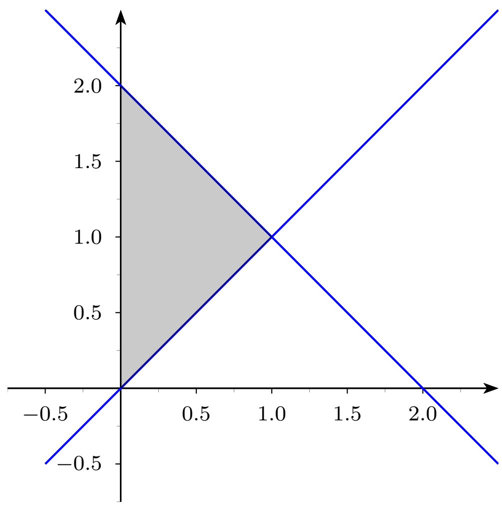
59. Sketch the region bounded by $y = 4 - x^2$ and $y = 3x$.



Hence,

$$\begin{aligned} V &= \int_{-4}^{1}\int_{3x}^{4-x^2}(x+4)dydx = \int_{-4}^{1}(x+4)(4-x^2-3x)dx \\ &= \int_{-4}^{1}(-x^3-7x^2-8x+16)dx = \left[-\frac{x^4}{4}-\frac{7x^3}{3}-4x^2+16x\right]_{-4}^{1} = \frac{625}{12} \end{aligned}$$

77. Sketch the region R.

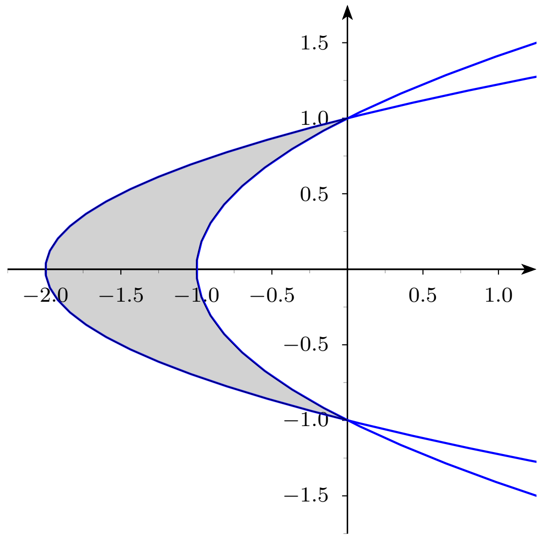


Reverse the order of integration,

$$\begin{aligned} V &= \int_{0}^{1}\int_{x}^{2-x}(x^2+y^2)dydx = \int_{0}^{1}(x^2(2-2x)+\frac{(2-x)^3-x^3}{3})dx \\ &= \int_{0}^{1}\left(-\frac{8}{3}x^3+4x^2-4x+\frac{8}{3}\right)dx = \frac{4}{3} \end{aligned}$$

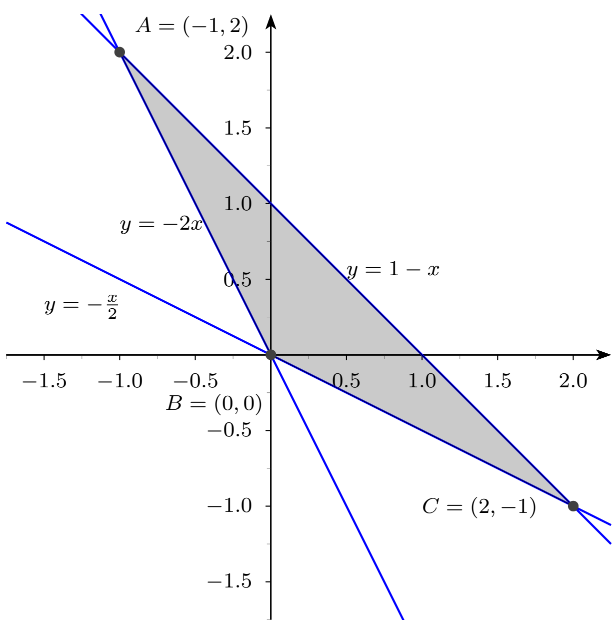
(15.3)

8. Sketch the region.



$$A = \int_{-1}^{1}\int_{2y^2-2}^{y^2-1} dxdy = \int_{-1}^{1}(1-y^2)dy = \frac{4}{3}$$

17. Sketch the region and find intersections



$$
\begin{aligned}
A &= \int_{-1}^{0}\int_{-2x}^{1-x} dydx + \int_{0}^{2}\int_{-x/2}^{1-x} dydx = \int_{-1}^{0}(1+x)dx + \int_{0}^{2}(1-\frac{x}{2})dx \\
&= \frac{1}{2} + 1 = \frac{3}{2}
\end{aligned}
$$

21. Area of region is $A = 2^2 = 4$, So the average height is

$$
\begin{aligned}
\text{Average height} &= \frac{1}{4}\int_{0}^{2}\int_{0}^{2}(x^2+y^2)dydx = \frac{1}{4}\int_{0}^{2}(2x^2+\frac{8}{3})dx \\
&= \frac{1}{4}\times\frac{32}{3} = \frac{8}{3}
\end{aligned}
$$